Journal of Applied Mathematics and Mechanics

ZAMM

Zeitschrift für Angewandte Mathematik und Mechanik
Founded by Richard von Mises in 1921

Edited in cooperation with Martin-Luther-Universität Halle-Wittenberg and Gesellschaft für Angewandte Mathematik und Mechanik e. V. (GAMM)

www.zamm-journal.org

WILEY

REPRINT

ZAMM · Z. Angew. Math. Mech. **92**, No. 3, 254 (2012) / **DOI** 10.1002/zamm.201100181

Comment

Authors' reply to "A remark to the paper by O. N. Kirillov and F. Verhulst "Paradoxes of dissipation-induced destabilization or who opened Whitney's umbrella?" [ZAMM 90, No. 6, 462–488 (2010)]

Received and accepted 16 December 2011
Published online 15 February 2012

We are glad to see from the remark that the paradoxical influence of small dissipation emphasized in the seminal work by Hans Ziegler of 1952 and resolved in 1956 by Oene Bottema continues to attract an attention of researchers, confirming that our efforts in writing the survey were not useless.

The remark states that the work by Bottema is "a study of the singularity in a simple specific case" while the authors of the remark "provided a final step in the qualitative understanding of Ziegler's paradox as a singularity". Note, however, that the "final step" was already taken by Bottema in 1956; since then it was re-iterated independently by a number of researchers.

Indeed, we quote from Bottema (1956) where he studies the stability of a *general* two-degrees-of-freedom non-conservative system with damping and compares its domain of asymptotic stability (A) with the domain of marginal stability in the absence of damping (B) in the space of the coefficients of the characteristic polynomial (a1, a2, a3); he directly points out that a1 and a3 depend on damping:

"One could expect B to be a limit of A, so that for $a1 \to 0$, $a3 \to 0$ the set A would continuously tend to B. That is not the case. . . . Here is the discontinuity we mentioned above. It plays a part in questions regarding the stability of equilibrium. The coefficients a1 and a3 depend on the linear damping forces and it is well known that the stability condition may change in a discontinuous way if a very small damping vanishes at all [1]. The phenomenon may be illustrated by a geometrical diagram. [1] Ziegler, Die Stabilitätskriterien der Elastomechanik, Ing. Arch. **20**, 49–56 (1952); Bottema, On the stability of the equilibrium of a linear mechanical system, Journal of Appl. Math. Phys. (ZAMP) **6**, 97–104 (1955)."

The geometrical diagram plotted by Bottema in 1956 is exactly the Whitney umbrella surface, one half of which bounds the domain of asymptotic stability. Bottema, an established geometer, correctly classifies it as a ruled surface and describes in detail its properties in terms of the generators. Therefore, it was Bottema who in 1956 resolved the Ziegler's paradox by discovering the Whitney umbrella singularity on the stability boundary of two-degrees-of freedom non-conservative systems that include Ziegler's pendulum as a particular case.

The results of Arnold on singularity theory of 1970s were quickly known in the Western dynamical systems community. For example, already in 1995, in the work [I. Hoveijn and M. Ruijgrok, The stability of parametrically forced coupled oscillators in sum resonance, Z. Angew. Math. Phys. **46**, 384–392 (1995)] the Arnold's method of versal deformation of matrix families was used in order to describe the destabilizing effect of damping in a general four dimensional dynamical system in sum resonance with application to rotating shafts. In the space of parameters of the versal deformation, Hoveijn and Ruijgrok found a generic Whitney umbrella singularity from the list of Arnold and plotted the corresponding singular stability threshold. Similar ideas were discussed in [S. A. van Gils, M. Krupa, and W. F. Langford, Hopf bifurcation with non-semisimple 1:1 resonance, Nonlinearity **3**, 825–850 (1990)]. Both papers precede the (in themselves valuable) references in the note by Seyranian and Mailybaev.